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ABSTRACT

The foreknowledge that a given school mathematics problem might have no specific solution or no solution at all affords students the challenge of investing careful consideration in the explicit problem conditions, as well as in the problem's unstated assumptions. Hence, the implication is that mathematics teachers, rather than speaking to their students in terms of solving problems, might better direct their students in terms of "resolving" problems, that is, either finding an acceptable solution or establishing why no solution exists. A much higher level of mathematical reasoning than that found in traditional classroom settings is required in making decisions about why there is no solution; perhaps, a still higher level of reasoning is necessary for students to modify the contents of such problems and provide for possible solutions. This problem resource book contains over 150 problems ranging over difficulty levels appropriate for grades 4-9. With about half the problems having no solution as written, the book is divided into four sections: area/volume, fractions, locus-of-points, and inverse operations. Each section contains answer keys for the teacher and suggestions for class presentation. (JJK)

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A TRICON MATHEMATICS
TEACHER'S RESOURCE MONOGRAPH

CAN IT BE?

A different PROBLEM RESOURCE BOOK

ABOUT HALF OF THE PROBLEMS
HAVE NO SOLUTION

THE STUDENT'S JOB IS TO FIND A SOLUTION
OR TO EXPLAIN WHY THERE IS NONE

AREA, VOLUME, PERIMETER
FRACTIONS OF SETS
GEOMETRY-LOCUS OF POINTS
INVERSE OPERATIONS

FOR GRADES 4 THROUGH 9

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A TRICON MATHEMATICS TEACHER'S RESOURCE MONOGRAPH

CAN IT BE?

WILLIAM L. SWART

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INTRODUCTION

Approximately half of the problems in this book do not have solutions. Knowing that some of the problems have no solutions, the student is challenged to give careful consideration to the stated conditions and to unstated assumptions. Moreover, a much higher level of reasoning is required in deciding why there is no solution, and a still-higher level to modify the given information, if possible, so that there is a solution.

It is suggested that you speak to your students about "resolving" problems, rather than solving them. A problem is resolved if all of its solutions are found, or if it is established that there is no solution.

The problems are intended to provide enrichment for all students. They range in difficulty from problems which can be resolved by almost all students at the mid-elementary level to those which will challenge the most capable middle-school students.

You can tailor your problems to age and individual ability in three ways:

(1) By assigning problems which you consider to be of the appropriate difficulty level.

(2) By the degree of rigor you require for an argument that a problem has no solution.

(3) By requiring your most capable students to change the conditions or the numbers in a no-solution problem to create a problem with a solution.

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AREA-VOLUME-PERIMETER

COMPLEXITY LEVELS AND SOLUTION METHODS FOR AREA-VOLUME PROBLEMS

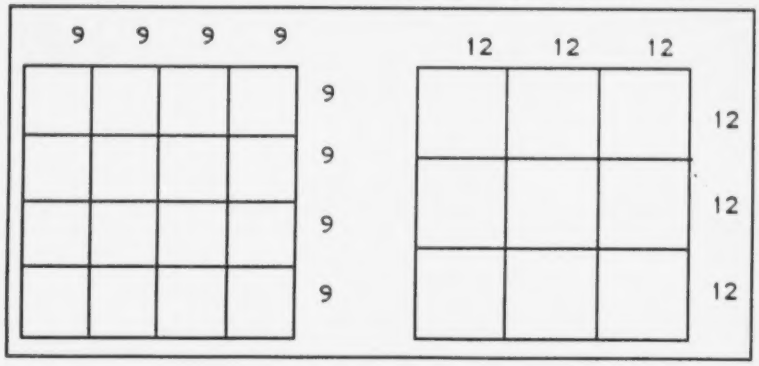
Because of the range of complexity in the area-volume section, each problem has been given a complexity rating from 1(least complex) to 3 (most complex). The solution method codes are: (G) for problems which can be resolved by diagrams drawn on graph paper, (F) for problems which can be resolved by formulas, (M) for using manipulatives, and (T) for problems which can be solved by making a table.

MATERIAL AND PREREQUISITE CONCEPTS FOR AREA-VOLUME PROBLEMS

(1) Cardboard squares.
(2) Graph paper. Concept that the area of a rectangle is the number of square units it takes to cover it.
(3 through 9) Graph paper.
(10) Cardboard squares.
(11) Meaning of perimeter.
(12) Graph paper.
(13 and 14) Graph paper. Meaning of area and perimeter.
(15) Graph paper.
(16) That the area of a rectangle is the number of square units to cover it.
(17) Graph paper. Meaning of area and perimeter.
NOTE: This exercise is designed to teach students that, for a given amount of perimeter (fence), the greatest rectangular area which can be formed is a square. This can make a good cooperative-learning problem. Have each student perform the task for one of the following perimeters, and then observe that in all cases a square maximizes the area. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40.
(18) Graph paper.
(19) For a given perimeter, the rectangle with maximum area is a square, the concept that is developed in prob. 17.
(20 through 22) Knowledge of the directions: north, southwest, etc.
(23) Graph paper.
(24) What a cube is.
(25 through 28) Graph paper.
(29) Some books of different sizes, to experiment.
(30) Some containers.
(31) What a cubic foot is.
(32) Formula for the area of a circle: A = Pi x radius squared.
(33) Graph paper.
(34 through 36) What a cube is.
(37) Graph paper. Can use Pythagorean theorem, but it's not necessary.
(38) Definition of equilateral triangle.
(39) Pythagorean theorem.
(40) Graph paper, formula for area of a circle.

SOLUTIONS

1. A 9-inch floor tile is broken. What is the least number of tiles which must be removed if the replacement is to be made with whole 12-inch tiles?
Complexity Level: 1 Solution Method: G,M.



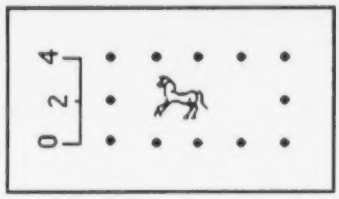
SOLUTION: Can it be? **Yes,** have the student cut out 9-inch squares of paper or cardboard and 12-inch squares, and lay out the squares to create a rectangle which can be covered by 9-inch squares and by 12-inch squares. If your classroom or hallway floor is covered with 9-inch or 12-inch tiles, then only one size must be cut out of paper to solve the problem. Sixteen tiles must be removed. At a higher level of reasoning, the sides of the rectangle must be divisible by 9 and by 12, so we want the least common multiple of 9 and 12 for each side, which is 36.

2. Given 18 feet of fence, draw a diagram of a rectangular horse pen which uses all of the fence (perimeter = 18 feet), which has whole-numbers of feet in its width and length, and which has area of 25 square feet.

SOLUTION: Can it be? **No,** given that the measures of sides are whole numbers, there is a finite number of possibilities. Any such rectangle we construct has less than 20 square feet of area. Here are all the possible pens with whole-number dimensions and a perimeter of 18. **Complexity Level: 1 Solution Method: G,F**

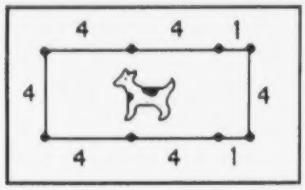
dimensions: width x length	area
1 x 8	8 sq ft
2 x 7	14 sq ft
3 x 6	18 sq ft
4 x 5	20 sq ft
5 x 4	(a repeat

3. A rectangular horse pen is 4 feet wide and 8 feet long. The fence posts are evenly spaced, 2 feet apart. How many posts are there?



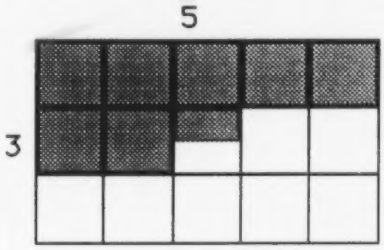
Complexity Level: 1 **SOLUTION:** Can it be? **Yes**, 12 posts.
Solution Method: G.

4. A rectangular dog pen is 4 feet wide and 9 feet long. The fence posts are evenly spaced, 4 feet apart. How many posts are there?
Complexity level: 1 Solution method: G



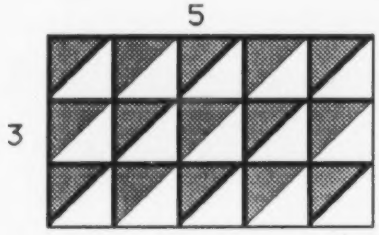
SOLUTION: Can it be? **No**, the sides are not divisible by 4.

5. Draw a rectangle of width 3 centimeters and length 5 centimeters. Draw lines inside the rectangle to make a square-centimeter grid and shade half of the centimeter squares. You must shade whole squares. If you shade a square, you must shade all of it.
Complexity Level: 1 Solution Method: G



SOLUTION: Can it be? **No.**

6. Draw a rectangle of width 3 centimeters and length 5 centimeters. Draw lines inside the rectangle to make a square-centimeter grid and shade half of each centimeter square. How many square centimeters are shaded?
Complexity Level: 1 Solution Method: G



SOLUTION: Can it be? **Yes**, 7 1/2 centimeters.

7. Draw 3 different rectangles with area 24. The sides must measure in whole numbers.
Complexity Level: 1 Solution Method: G

SOLUTION: Can it be? **Yes.** 1 x 24, 2 x 12, 3 x 8

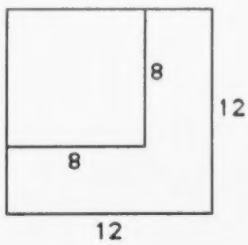
8. Draw 3 different rectangles with area 15. The sides must measure in whole-numbers. **Complexity Level: 1**
Solution Method: G

SOLUTION: Can it be? **No.** Only a 1 x 15 and a 3 x 5 rectangle meet the criteria.

9. Draw 3 different rectangles with area 13. The sides must measure in whole numbers.
Complexity Level: 1 Solution Method: G

SOLUTION: Can it be? **No.** Since 13 is a prime number, only one such rectangle exists.

10. A 12-inch floor tile is broken. How many 8-inch tiles will it take to replace the 12-inch tile without cutting the 8-inch tiles?
Complexity Level: 1 Solution Method: G



SOLUTION: Can it be? **No.**

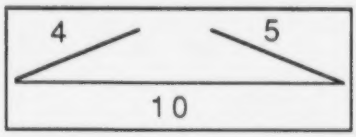
11. Draw as many rectangles as possible which have perimeter of 12, allowing the fraction 1/2 in the measures of the sides.
Complexity Level: 1 Solution Method: G, T

SOLUTION: Can it be? **Yes**, the rectangles listed here. This problem can lead to consideration of precision. Does a measure such as 2.0 involve the fraction 1/2? It does because zero tenths implies measurement to the nearest tenth, and zero tenths implies zero halves.

0.5	by	5.5
1.0	by	5.0
1.5	by	4.5
2.0	by	4.0
2.5	by	3.5
3.0	by	3.0
3.5	by	2.5(a repeat)

12. What is the area of a triangle with sides of 4 centimeters, 5 centimeters , and 10 centimeters? **Complexity Level: 1 Solution Method: G**

SOLUTION: Can it be? **No**, the sum of the lengths of the two shorter sides must be greater than the longer side.



13. Draw as many rectangles as possible, with whole-number widths and lengths, perimeter of 16, and area of 12.
Complexity Level: 1 Solution Method: G, T

SOLUTION: Can it be? **Yes**, Draw the rectangles on graph paper:

Width x length	area	OK?
1 x 7	7	no
2 x 6	12	yes
3 x 5	15	no
4 x 4	16	no
5 x 3	a repeat	

14. Draw as many rectangles as possible, having whole-number widths and lengths, with perimeter of 12, and area of 12. **Complexity Level: 1 Solution Method: G, T**

SOLUTION: Can it be? **No**, draw the rectangles on graph paper:

width x length	area	area = 12?
1 x 5 =	5	no
2 x 4 =	8	no
3 x 3 =	9	no

15. Draw as many rectangles as possible with whole-number widths and lengths, having area 24 square units. **SOLUTION**: Can it be? **Yes**, see table.
Complexity Level: 1 Solution Method: G, F.

width x length	area
1 x 24 =	24
2 x 12 =	24
3 x 8 =	24

16. Draw as many rectangles as possible with whole-number widths and lengths, having area 29 square units. **Complexity Level: 1 Solution Method: G, F.**

SOLUTION: Can it be? **Yes**, 1 x 29. Only one case, since 29 is a prime number.

17. This is a simple problem, but rather lengthy. Read the entire problem before you start, so you can plan the work. You have 16 feet of fence. (A) Draw on graph paper all of the rectangular dog pens which can be built with 16 feet of fence, having whole number lengths and widths. Make a table for the pens like this:

Pen No.	Width	Length	Area
1	1	7	7
2	2	etc.	etc.
etc.	etc.		

(B) Draw all of the rectangles on graph paper for 20 feet of fence as you did for 16 feet in part A of this problem, and make the same kind of table as you did for part A.

(C) Repeat the process for 24 feet of fence.

Study the tables you made to answer these questions.
(D) If I have 28 feet of fence what dimensions should I use to maximize (have the most) area in the pen?

(E) What dimensions should I use for 36 feet of fence?

(F) For any amount of fence, what kind of rectangle would you advise me to make to maximize the area?

NOTE: This is a good cooperative-learning problem. Assign certain individuals the job of making the table for a given number. Then have them get together and discuss the implications of their tables for question D.

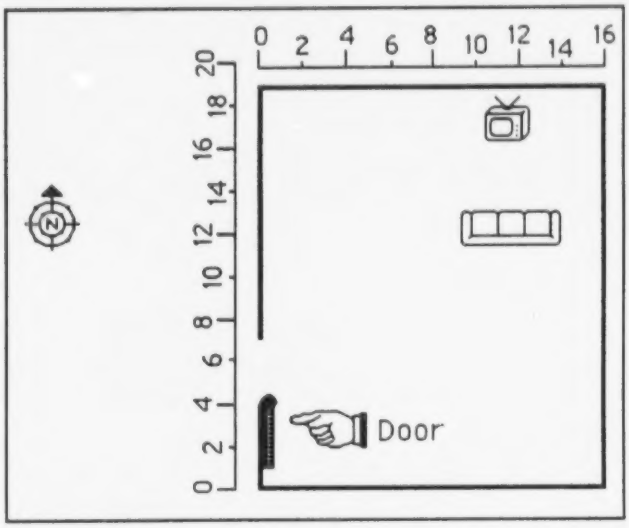
18. Draw a diagram of a goat pen which will have area of 800 square meters, using no more than 100 meters of fence wire. **Complexity Level: 2 Solution Method: G.**

SOLUTION: Can it be? **No**, as problem 17 shows, a square makes the rectangle with greatest area for a given amount of perimeter. Using 100 feet of fence to make a square produces a 25 x 25 foot square, with area of 625.

19. Given 18 feet of fence, draw a diagram of a zebra pen which uses all of the fence, and which has area of 36 square feet. **Complexity Level: 2**

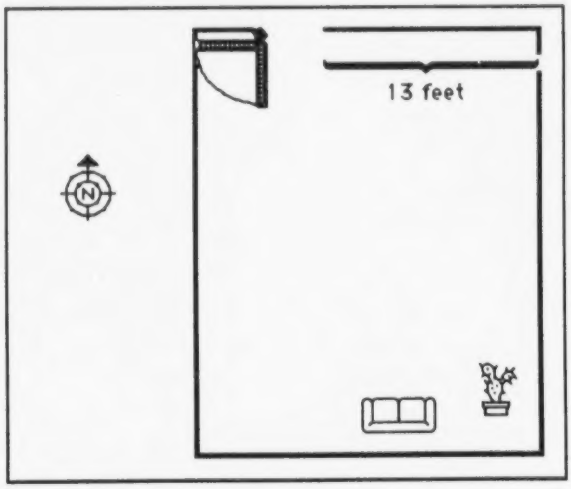
SOLUTION: Can it be? **No**, we can't list all possibilities. Since the measures of the sides need not be whole numbers, there is an infinite number of possibilities: 4 1/4 by 4 3/4, 4 1/4 by 4 3/4, 5 by 4, 1 1/2 by 7 1/2, etc. However, for a given amount of fence (perimeter) the maximum rectangular area is a square. A square with perimeter of 18 has sides of 4.5 and area of 4.5 x 4.5 = 20.25.

20. The door on the west side of a square bedroom is 3 feet wide, and swings into the room. The south edge of the door is 4 feet from the south-west corner of the room. The north edge of the door is 12 feet from the north-west corner of the room. How much wall-space for furniture is lost by that placement of the door? **Complexity Level: 1**
Solution Method: G.



SOLUTION: Can it be? **Yes**, three feet, if we want the door to swing 180 degrees all the way to the wall.

21. The north side of a rectangular bedroom is 16 feet long. The west side is 20 feet long. The door is on the north side, is 3 feet wide, and swings into the room. The hinges are on the west side of the door. The east edge of the door itself is 13 feet from the northeast corner of the room. How much wall space for furniture is lost by that placement of the door? **Complexity Level: 1**
Solution Method: G



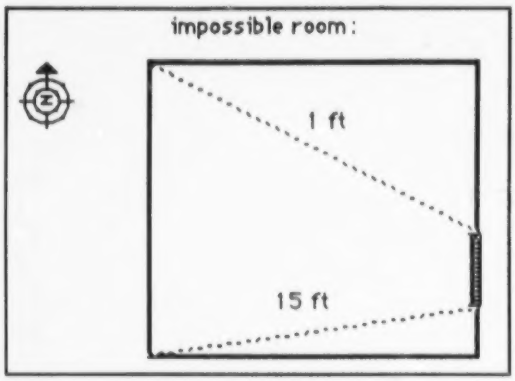
SOLUTION: Can it be? **Yes**, three feet

If the door is to swing only 90 degrees, no wall space is lost. If the door is to swing 180 degrees, then three feet on the north wall is lost plus an indeterminate amount on the west wall, outside of the circle traced by the edge of the door.

22. The door on the east side of a bedroom is 3 feet wide, and swings into the room. The south edge of the door is 15 feet from the southwest corner of the room. The north edge of the door is 1 foot from the northwest corner of the room. How much wall space for furniture is lost by that placement of the door?

Complexity Level: 2
Solution Method: G

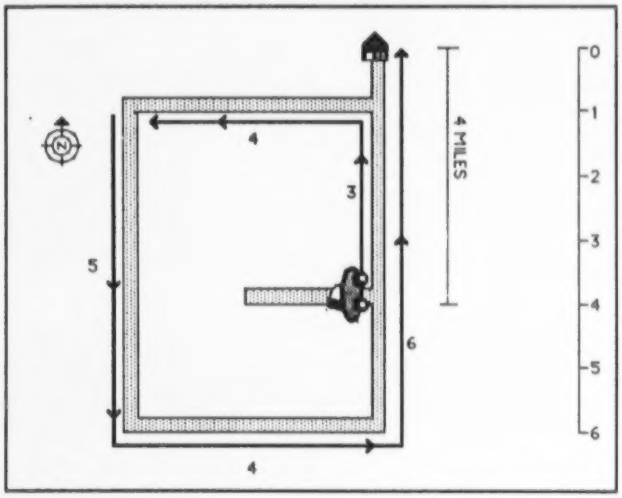
SOLUTION: Can it be? **No,** the conditions lead to a bedroom with the dimensions shown in the diagram.



23. A car started at the intersection of two roads. It traveled 3 miles north, and turned west. It traveled 4 miles west, and turned south. It traveled 5 miles south, and turned east. It traveled 4 miles east, and turned north. It traveled 6 miles north. How far is the car from its starting point?

Complexity Level: 1
Solution Method: G

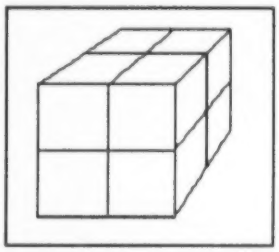
SOLUTION: Can it be? **Yes,** 4 miles.



24. What is the smallest number of cubes which can be used to build a larger cube?

Complexity Level: 1
Solution Method: M

SOLUTION: Can it be? **Yes,** 8 cubes.



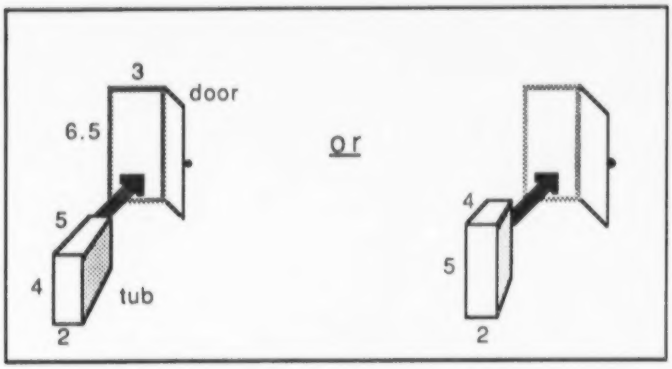
25. You have 20 feet of fence to build a chicken pen, and you want each chicken to have 2 square feet of area in the pen. You have 30 chickens. How much of the fencing must you use to build a rectangular pen for 30 chickens? Use graph paper to draw the pen. **Complexity Level: 2 Solution Method: GF**

SOLUTION: Can it be? **No** a square pen with sides of 5 feet has only 25 square feet of area. Problem number 17 shows that, for a given perimeter, a square maximizes area.

26. The door on Willy's house is 3 feet wide and 6.5 feet high. Willy has a bath tub which is 4 feet wide, 5 feet long, and 2 feet high. Can Willy get the tub through the door?

Complexity Level: 1
Solution Method: G.

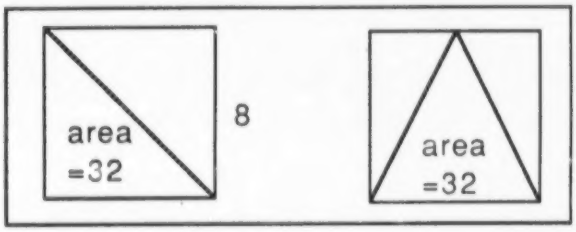
SOLUTION: Can he? **Yes.**



27. Draw a square with sides of 8 centimeters. Draw a triangle. inside the square which has area 40 square centimeters.

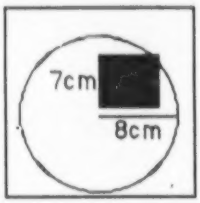
Complexity Level: 1
Solution Method: G

SOLUTION: Can it be? **No.**



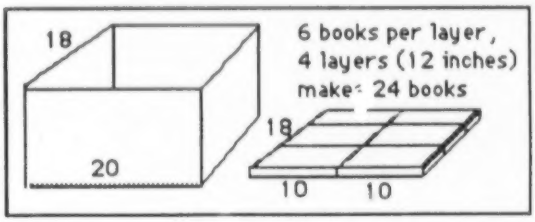
28. Consider a square wooden surface with area 49 square centimeters. Suppose you drilled a circular hole in the surface with radius 8 centimeters. If a container of paint will cover 4 square centimeters, how many of those containers of paint will it take to cover the remaining surface? **Complexity Level: 1 Solution Method: G, F.**

SOLUTION: Can it be? **No,** the problem is simple to resolve by drawing a square on graph paper and drawing a circle on the square. The diameter of the hole to be drilled is greater than the diagonal of the square. The diagonal = the square root of 98, or about 9 9



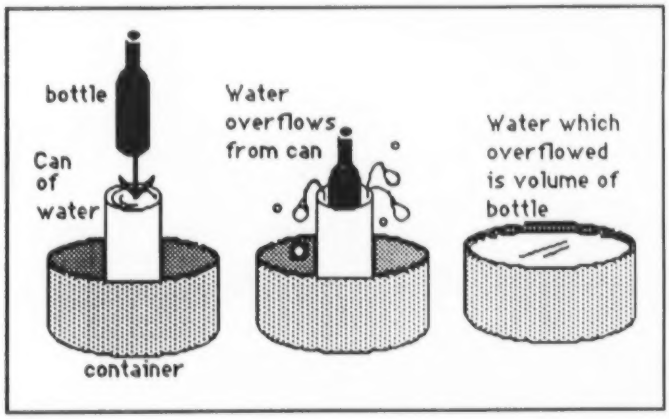
29. Jill works for a printing company. She has been given the job of ordering boxes to ship dictionaries. The books are 3 inches thick, six inches wide, 10 inches long, and weigh 4 pounds. She wants to have boxes made which will hold 2 dozen books, with no wasted space in the box. No dimension of the boxes can be greater than 36 inches. She should order boxes which are how wide, how long, and how high? **Complexity Level: 2 Solution Method: M**

SOLUTION: Can it be done? **Yes, there is at least one solution.** (use books which are close to 3 x 10 to experiment)



30. You can find the volume (space occupied) by an irregular-shaped object by the following method: Place a can (A) inside another container (B). Fill the can with water, and carefully lower the object into the can. The volume of water which flows out of the can into the container is approximately equal to the volume of the object. Get a small glass or plastic bottle, and use the method to find the approximate volume of plastic or glass which was used to make the bottle. **Complexity Level: 2**
Solution Method: M

SOLUTION: Can it be done? **Yes,** Immerse the open bottle in can A. The water which overflows into container B equals the volume of glass or plastic used to make the bottle.



31. You have 400 pounds of wet sand which weighs 50 pounds per cubic foot. You have boxes which hold 1.5 cubic feet of the sand when full. How many of those boxes will it take to hold the sand, if each box is to be full? **Complexity Level: 2 Solution Method: T**

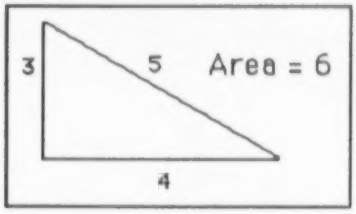
Boxes	Cubic Feet
1	1.5
2	3
3	4.5
4	6
5	7.5
6	9

SOLUTION: Can it be? **No**. Dividing 400 by 50, we have 8 cubic feet of sand. Dividing 8 by 1.5 , we get 5.3..., which means we cannot fill a whole number of boxes with the sand.

32. Given a circle with radius 3 centimeters, find the width and length of a rectangle inside the circle, with each vertex on the circle, and area of 40 square centimeters. **Complexity Level: 1 Solution Method: G, F**

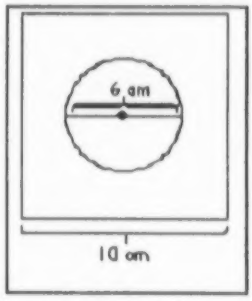
SOLUTION: Can it be? **No**, the circle area is less than 40 square centimeters. Since the circle's area = 9π, and $\pi < 4$, the circle's area is less than 36. Graph paper proof is found by drawing a square with sides tangent to the circle, and observing that it has area of only 26 square centimeters:

33. What is the area of a triangle with sides of 3 centimeters, 4 centimeters, and 5 centimeters? **Complexity Level: 1 Solution Method: G, F.**



SOLUTION: Can it be? **Yes**, it is a right triangle, which can be established by drawing on graph paper or by observing that 3, 4, 5, is a Pythagorean triple. A graph-paper drawing will show the area to be 6.

34. How many spherical balls of radius 3 centimeters can be placed in a cubic box with volume of one liter? **Complexity Level: 1 Solution Method: M, G.**

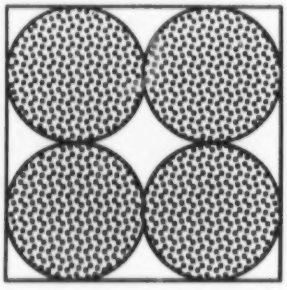


SOLUTION: Can it be? **Yes,** as the drawing shows, one ball will fit.

35. How many spherical balls of radius 5 centimeters can be placed in a cubic box having volume of one liter? **Complexity Level: 1 Solution Method: G, A.**

SOLUTION: Can it be? **Yes**, on a naive level, one ball (tight fit).

36. How many spherical balls of radius 2.5 can be placed in a cubic box with volume of one liter? **Complexity Level: 1 Solution Method: G**



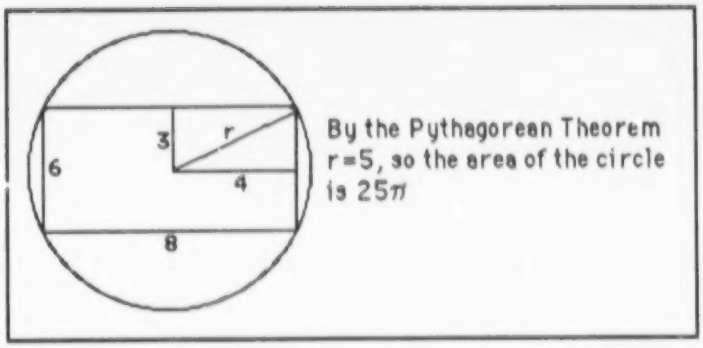
SOLUTION: Can it be? **Yes**, eight balls. This is essentially the same question as how many 2.5-inch cubes would fit in the box.

However, as in problem 15, as a practical matter, the diameter of each ball would have to be slightly less than the diameter given here.

37. A rectangle of width 6 and length 8 is drawn inside a circle. Each vertex of the rectangle falls on the circle. What is the area of the circle? **Complexity Level: 2 Solution Method: G, F**

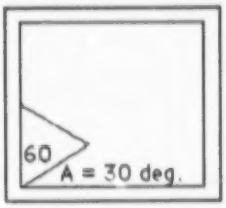
SOLUTION: Can it be? **Yes**, using graph paper: Draw the rectangle and draw the circle around it. Then estimate the circle's area by counting square units on the graph paper.

Using Formulas: By the Pythagorean theorem, r =5, so the area of the circle is 25π.



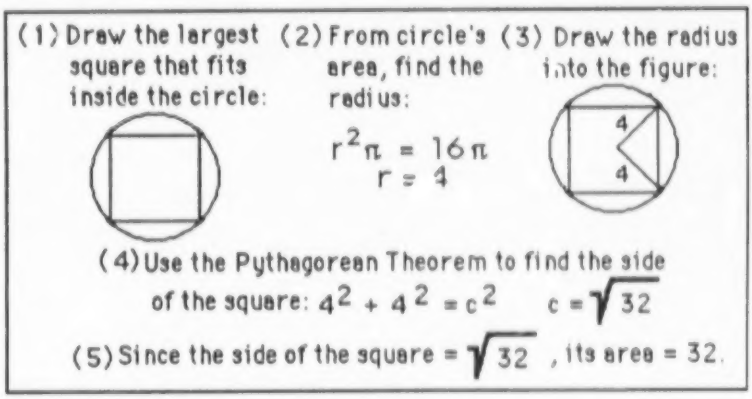
38. How many paper equilateral triangles with sides of 1 inch does it take to cover a square with sides of 4 inches? **Complexity Level: 3 Solution Method: G**

SOLUTION: Can it be? **No**, an equilateral triangle has angles of 60 degrees, and angle A cannot be filled by a paper triangle with angles of 60 degrees.



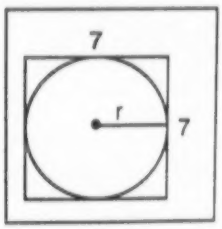
39. Given a circle with area of 16π, what is the area of the greatest square which can be drawn inside the circle? **Complexity Level: 3 Solution Method: G,F**

SOLUTION: Can it be? **Yes**, we need the Pythagorean theorem.


(1) Draw the largest square that fits inside the circle:
(2) From circle's area, find the radius:
$r^2\pi = 16\pi$
$r = 4$
(3) Draw the radius into the figure:
4
4
(4) Use the Pythagorean Theorem to find the side of the square: $4^2 + 4^2 = c^2$ $c = \sqrt{32}$
(5) Since the side of the square = $\sqrt{32}$, its area = 32.

40. Given a square with area 49 square centimeters, what is the area of the greatest circle which can be drawn inside the square? **Complexity Level: 3 Solution Method: G,F**

SOLUTION: Can it be? **Yes.** If we define "inside" to allow the sides of the square to be tangent to the circle, then the radius is 3 1/2 centimeters, half the length of the side, and the circle's area is 49/4π square centimeters. However, this problem can lead to the non-trivial question of what it means for a circle to be inside the square. If the circle is not to touch the sides of the square, then its radius must be some amount (call amount E) less than 3 1/2 centimeters. But how much less? An E-value of one millimeter will keep the circle from touching the side. So will an E-value of half a millimeter. So will E=0.0000000005 of a millimeter. This is a good problem for discussion of the limit concept. For any E-value, however small, there is an E-value even smaller, which will also do the job of preventing the circle from touching the side of the square. So, if we do not allow the sides to be tangent to the circle, then there is no largest circle which can be drawn inside the square. There is an upper limit, of course, the circle with radius 3 1/2.



AREA-VOLUME-PERIMETER PROBLEMS

1. A 9-inch floor tile is broken. What is the least number of tiles which must be removed if the replacement is to be made with whole 12-inch tiles? Can it be?

2. Given 18 feet of fence, draw a diagram of a rectangular horse pen which uses all of the fence (perimeter = 18 feet), which has whole-numbers of feet in its width and length, and which has area of 25 square feet. Can it be?

3. A rectangular horse pen is 4 feet wide and 8 feet long. The fence posts are evenly spaced, 4 feet apart. How many posts are there? Can it be?

4. A rectangular dog pen is 4 feet wide and 9 feet long. The fence posts are evenly spaced, 4 feet apart. How many posts are there? Can it be?

5. Draw a rectangle of width 3 centimeters and length 5 centimeters. Draw lines inside the rectangle to make a square-centimeter grid and shade half of the centimeter squares. You must shade whole squares. If you shade a square, you must shade all of it. Can it be?

6. Draw a rectangle of width 3 centimeters and length 5 centimeters. Draw lines inside the rectangle to make a square-centimeter grid and shade half of each centimeter square. How many square centimeters are shaded? Can it be?

7. Draw 3 different rectangles with area 24. The sides must measure in whole numbers. Can it be?

8. Draw 3 different rectangles with area 15. The sides must measure in whole-numbers. Can it be?

9. Draw 3 different rectangles with area 13. The sides must measure in whole numbers. Can it be?

10. A 12-inch floor tile is broken. How many 8-inch tiles will it take to replace the 12-inch tile without cutting the 8-inch tiles? Can it be?

11. Draw as many rectangles as possible which have perimeter of 12, allowing the fraction 1/2 in the measures of the sides. Can it be?

12. What is the area of a triangle with sides of 4 centimeters, 5 centimeters , and 10 centimeters?

13. Draw as many rectangles as possible, with whole-number widths and lengths, perimeter of 16, and area of 12. Can it be?

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14. Draw as many rectangles as possible, having whole-number widths and lengths, with perimeter of 12, and area of 12.

15. Draw as many rectangles as possible with whole-number widths and lengths, having area 24 square units. Can it be?

16. Draw as many rectangles as possible with whole-number widths and lengths, having area 29 square units. Can it be?

17.(*Teachers , see the note in your copy regarding group activity for this problem*) You have 16 feet of fence. (A) Draw on graph paper all of the rectangular dog pens which can be built with 16 feet of fence, having whole number lengths and widths. Make a table for the pens like this:

Pen No.	Width	Length	Area
1	1	7	7
2	2	etc.	etc.
etc.	etc.		

(B) Draw all of the rectangles on graph paper for 20 feet of fence as you did for 16 feet in part A of this problem, and make the same kind of table as you did for part A.

(C) Repeat the process for 24 feet of fence.

Study the tables you made to answer these questions.

(D) If I have 28 feet of fence what dimensions should I use to maximize (have the most) area in the pen?

(E) What dimensions should I use for 36 feet of fence?

(F) For any amount of fence, what kind of rectangle would you advise me to make to maximize the area?

18. Draw a diagram of a goat pen which will have area of 800 square meters, using no more than 100 meters of fence wire. Can it be?

19. Given 18 feet of fence, draw a diagram of a zebra pen which uses all of the fence, and which has area of 36 square feet. Can it be?

20. The door on the west side of a square bedroom is 3 feet wide, and swings into the room. The south edge of the door is 4 feet from the south-west corner of the room. The north edge of the door is 12 feet from the north-west corner of the room. How much wall-space for furniture is lost by that placement of the door? Can it be?

21. The north side of a rectangular bedroom is 16 feet long. The west side is 20 feet long. The door is on the north side, is 3 feet wide, and swings into the room. The hinges are on the west side of the door. The east edge of the door itself is 13 feet from the northeast corner of the room. How much wall space for furniture is lost by that placement of the door?

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22. The door on the east side of a bedroom is 3 feet wide, and swings into the room. The south edge of the door is 15 feet from the south- west corner of the room. The north edge of the door is 1 foot from the north-west corner of the room. How much wall space for furniture is lost by that placement of the door? Can it be?

23. A car started at the intersection of two roads. It traveled 3 miles north, and turned west. It traveled 4 miles west, and turned south. It traveled 5 miles south, and turned east. It traveled 4 miles east, and turned north. It traveled 6 miles north. How far is the car from its starting point? Can it be?

24. What is the smallest number of cubes which can be used to build a larger cube?

25. You have 20 feet of fence to build a chicken pen. You want each chicken to have 2 square feet of area in the pen. You have 30 chickens. How much of the fence must you use to build a rectangular pen for 30 chickens? Use graph paper to draw the pen. Can it be?

26. The door on Willy's house is 3 feet wide and 6.5 feet high. Willy has a bath tub which is 4 feet wide, 5 feet long, and 2 feet high. Can Willy get the tub through the door?

27. Draw a square with sides of 8 centimeters. Draw a triangle inside the square which has area 40 square centimeters. Can it be?

28. Consider a square wooden surface with area 49 square centimeters. Suppose you drilled a circular hole in the surface with radius 8 centimeters. If a container of paint will cover 4 square centimeters, how many of those containers of paint will it take to cover the remaining surface? Can it be?

29. Jill works for a printing company. She has been given the job of ordering boxes to ship dictionaries. The books are 3 inches thick, six inches wide, 10 inches long, and weigh 4 pounds. She wants to have boxes made which will hold 2 dozen books, with no wasted space in the box. No dimension of the boxes can be greater than 36 inches. She should order boxes which are how wide, how long, and how high? Can it be done?

30. You can find the volume (space occupied) by an irregular-shaped object by the following method: Place a can (A) inside another container (B). Fill the can with water, and care fully lower the object into the can. The volume of water which flows out of the can into the container is approximately equal to the volume of the object. Get a small glass or plastic bottle, and use the method to find the approximate volume of plastic or glass which was used to make the bottle. Can it be?

31. You have 400 pounds of wet sand which weighs 50 pounds per cubic foot. You have boxes which hold 1.5 cubic feet of the sand when full. How many of those boxes will it take to hold the sand, if each box is to be full? Can it be?

32. Given a circle with radius 3 centimeters, find the width and length of a rectangle inside the circle, with each vertex on the circle, and area of 40 square centimeters. Can it be?

33. What is the area of a triangle with sides of 3 centimeters, 4 centimeters, and 5 centimeters? Can it be?

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34. How many spherical balls of radius 3 centimeters can be placed in a cubic box with volume of one liter? Can it be?

35. How many spherical balls of radius 5 centimeters can be placed in a cubic box having volume of one liter? Can it be?

36. How many spherical balls of radius 2.5 can be placed in a cubic box of one liter volume? Can it be?

37. A rectangle of width 6 and length 8 is drawn inside a circle. Each vertex of the rectangle falls on the circle. What is the area of the circle?

38. How many paper equilateral triangles with sides of 1 inch does it take to cover a square with sides of 4 inches? Can it be?

39. Given a circle with area of 16π, what is the area of the greatest square which can be drawn inside the circle? Can it be?

40. Given a square with area 49 square centimeters, what is the area of the greatest circle which can be drawn inside the square? Can it be?

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FRACTIONS SECTION

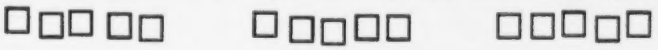
TEACHING SUGGESTIONS FOR THE FRACTIONS SECTION

For these problems your students need a good foundation in fractions of sets. The commonplace work with rectangular regions and circles will not adequately prepare them. The following three objectives are recommended as prerequisites to the Can-It-Be fraction exercises.

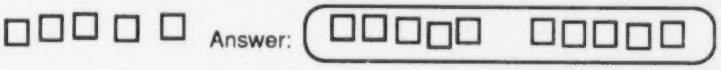
OBJECTIVE 1: Finding a fraction of a set. First, have the student establish an "Answer Place" on the desk. That can be a container or simply a piece of paper with "Answer Place" written on it. To do the exercise 2/3 of 15:

Step 1: Count out 15 blocks or other counters.

Step 2: Consider the two numbers in a fraction to be messengers. The message carried by the bottom number of the fraction (Mr. D, if you like) is to arrange the 15 objects into 3 equal piles.



Step 3: The message of the top number (Mr. N) is to move 2 of those piles to the answer place.



Note: While Objective 1 is learned quickly by virtually all students from grade 4 on up, it takes quite a while for some of them to get the "Famous Fraction Error" out of their systems. That's the error of counting the blocks which were <u>not</u> moved to the Answer Place, rather than those which <u>were</u> moved. That might be because the final action in taking a fraction of a set, the moving of blocks, is similar to the action performed in subtraction, where the answer is the remainder, rather than what is moved. Ordinarily we don't teach students about errors, but this one is so pervasive that talking about it when introducing the objective seems to help cope with it.

OBJECTIVE 2: Finding a missing numerator.
Finding missing numerators with blocks is fairly simple. For the exercise ?/5 of 20 = 12, we speak of it as an exercise which someone performed, the answer was 12, and the numerator has been lost (coffee spilled on it).

Step 1: Count out 20 blocks, and put them in 5 piles.

Step 2: How many piles must be moved to the Answer Place to have 12 blocks there?

OBJECTIVE 3: Finding missing denominators.
Finding missing denominators is not so simple. There are two ways to do it, analytically, or by guess-and-check. It is strongly recommended that you <u>not</u> teach the analytical process to any but the most able and perceptive students. The guess-and-check method can be learned by all, and contributes greatly to overall understanding of fractions of sets. To do 3/? x 36 = 27 by Guess and Check: Count out 36 cubes, and experiment. For example, try 6 as the denominator. But 6 doesn't work because 3/6 x 36 results in 18 blocks in the answer place, so try another number.

Note: Guess-and-check is one of the most useful problem solving methods, but one of the most difficult to get students to use. They seem to think that we don't really want them to guess, but somehow to manufacture a correct result mentally. When the student says "I don't know what to try," a good response is to say "Try your shoe size, today's date, your age, or your street address." In using guess-and-check on these and other problems it is a good idea to require them to record their guesses. Not only does that encourage them to use the process, but seeing their guesses will indicate to you how much insight they have into the problem.

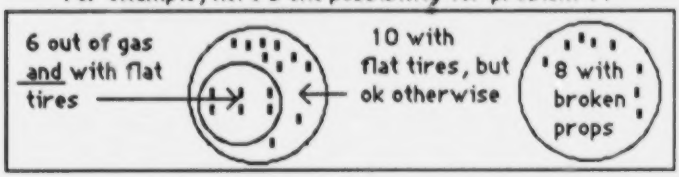
Exercise bank for the above objectives. For an exercise, N/D x X = __, pick a numerator from the top, a denominator from the left column, and a set-number from the body of the table.

		NUMERATORS					
		1	2	3	4	5	6
	2	16,18					
	3	12,24	12,30				
DENOMINATORS	4	16,36	14,24	12,36			
	5	20,25	15,25	20,25	15,30		
	6	12,24	18,36	12,24	24,36	24,30	
	7	14,21	14,21	21,28	21,28	14,28	21,28

FRACTION ANSWER KEY

It is recommended that you teach your students to use Venn diagrams to determine whether or not certain sets.

For example, here's one possibility for problem 7:



1. There were 36 coins on the table. 3/4 of them were pennies. 1/3 of them were dimes. 1/3 of them were quarters. How many pennies? Dimes? Quarters? Can it be? **SOLUTION: No,** 27 pennies +12 dimes + 12 quarters = 51 coins in all.

2. Each club member took 24 tickets to sell for the play. Jack sold 2/3 of his tickets. Jill sold 3/4 of her tickets. Dan sold 4/6 of his tickets. Betty sold 5/8 of her tickets. Jane sold 7/8 of her tickets. How many did each member sell? Can it be? **SOLUTION: Yes**, each student had 24 tickets and sold less than that.

3. Cathy broke a window playing ball. She agreed to work 24 hours to pay for the window. She made the following work plan: She planned to work 1/4 of the hours on Monday, 3/8 of the hours on Tuesday, 2/4 of the hours on Wednesday. After she works those 3 days, how many more days will she have to work? Can it be? **SOLUTION: No**, 1/4 of the 24 hours = 6, 3/8 of the 24 hours = 9, 2/4 of the hours = 12, for a total of 27.

4. Jack was supposed to do 19 whole push-ups for exercise. He said he did 2/3 of them before falling on his face. Jill said Jack made a mistake in his arithmetic. Why did Jill think so? Can it be? **SOLUTION: No**, 2/3 of 19 is 12.66.

5. There were 21 horses to be bathed. Mary bathed 1/3 of them. Jill bathed 2/7 of them. How many did Mary bathe? How many did Jill bathe? How many were still dirty? Can it be? **SOLUTION: Yes**, Mary bathed 7, Jill bathed 6, and 8 were still dirty.

6. BE SURE YOU LAY OUT THE BLOCKS AND COUNT THEM VERY CAREFULLY FOR THIS PROBLEM: Suppose you have 64 dogs and cats. 3/8 of them are black dogs. 1/4 of them are yellow cats. 4/8 of them are white cats. The rest are white dogs. How many are black dogs? Yellow cats? White cats? White dogs? Can it be? **SOLUTION: No**, the sets are disjoint (no members in common), and 24 + 16 + 32 = 72 animals.

7. There were 24 airplanes at the airport. 1/4 of them were out of gas. 2/3 of them had flat tires. 1/6 of them had broken propellers, and the rest were o.k. How many were o.k.? Can it be? **SOLUTION: Yes,** (1/4 X 24) 6 were out of gas. (2/3 x 24) 16 had flat tires. (1/6 x 24) 4 had broken propellers. **These sets can intersect**. It could be that 6 of the 16 with flat tires are out of gas and that 4 of that 16 have broken propellers, leaving 8 ready to fly. Or it could be that all 24 are disables. From 0 to 8 are ok. If each plane could carry 10 people, how many people could the o.k. planes carry? 0 to 80.

8. FOR THIS PROBLEM PRETEND EACH BLOCK IS 100 PEOPLE. There were 1800 people at the fair. 2/6 of them were Canadian, 1/3 of them were Mexican, 1/6 of them were African, and the rest were Chinese. How many were Canadian? Mexican? African? Chinese? Can it be? **SOLUTION: Yes**, 600 Canadian, 600 Mexican, 300 African, 300 Chinese totals 1800.

9. Mrs. Sanders won $24,000 in the lottery. Let each block represent 1,000 dollars. She spent 3/8 of it for a car, 1/4 of it to paint the house, and put the rest in the bank. Can it be? **SOLUTION: Yes**, $9,000 for the car, $6,000 for the paint, and $9,000 in the bank totals $24,000.

10. Binky invited 12 friends to a party. 1/3 of them came to the party. 1/2 of them ate cake. 4/6 of them ate ice cream. How many came? Ate cake? Ate ice cream? Can it be? **SOLUTION: No**, only 4 came, and 4/6 of 4 is not a whole number of friends.

11. Mrs. Smith has 12 children. 1/2 of them want to be fire fighters when they grow up. 1/2 of them want to be writers. 2/3 of them are little girls. How many want to be fire fighters? Writers? How many are girls? Can you be sure some of the girls want to be fire fighters? Can you be sure some of the girls want to be writers? Can it be? **SOLUTION: Yes**, 6 want to be fire fighters, 6 want to be writers, and 8 are girls. Yes, since more than 6 are girls, some girls want to be fire fighters, and some girls want to be writers.

12. 27 peaches grow on your peach tree. 1/3 of them are ripe. 2/3 of them are not ripe. 1/9 of them have holes in them. How many are ripe? Not ripe? How many have holes? Can it be? **SOLUTION: Yes**, the sets are not necessarily disjoint. Some peaches can be ripe and have holes.

13. There are 30 microscopes in a laboratory. 1/5 of them are for looking at bugs only. 1/2 of them are for looking at germs only. 2/3 of them are for looking at plants only. Can it be? **SOLUTION: No**, 6 + 15 + 20 = 41 microscopes. The lab has only 30.

14. Suppose there are 45 space ships circling the earth. 4/5 of them American, 1/3 of them Russian, 1/3 of them European. How many are American? Russian? European? Can it be? **SOLUTION: No**, 4/5 (36) of them are American, leaving only 9 to be European and/or Russian.

15. There are 30 students in the first grade. 1/5 of them are girls. 9/10 of them are boys. 1/3 of them walk to school. How many are girls? Boys? Walk to school? Can it be? **SOLUTION: No**, 6 + 27 = 31 students.

16. Gerald has 6 children. On his way to work, he drops 2/3 of them off at nursery school, and 1/2 of them off at an elementary school. How many go to nursery school? Elementary school? Can it be? **SOLUTION:** No, assuming the sets are disjoint (no child goes to both schools), 4 going to nursery school and 3 to elementary school = 7.

17. Don had 4 dreams last night. 1/2 were about cars. 3/4 were about his friends. 1/4 were about his parents. How many were about cars? Friends? Parents? Can it be? **SOLUTION: Yes**, the sets are not necessarily disjoint. A dream can be about friends, and cars and parents.

18. Betsy is building a house worth $12,000. 1/2 of that cost is for wood. 1/4 of that cost is for cement. 2/6 of that cost is for metal. How much for wood? Cement? Metal? Can it be? **SOLUTION: No**, the sets are disjoint, and 6,000 + 3,000 + 4,000 = 13,000

19. The monster motel houses 35 monsters. 2/3 of them are hairy. 4/7 of them have horns. 1/2 of them have three eyes. How many are hairy? Have horns? Have 3 eyes? Can it be? **SOLUTION: No**, 14 are hairy, 20 have horns, but 17.5 cannot have three eyes.

20. There are 40 bigfoot monsters on earth. 1/4 of them live in the arctic. 1/2 of them live at the equator. 2/4 of them live in the U.S.A. How many live in the arctic? At the equator? In the USA? Can it be? **SOLUTION: Yes**, the sets are not necessarily disjoint. Twenty of them could live in the USA and in arctic Alaska.

21. There were 36 people on an international committee. 1/3 of them were Americans, 1/4 of them were Iraqis, and 1/2 of them were Chinese. How many of them were Americans? Iraqis? Chinese? Can it be? **SOLUTION: Yes**, if some of the members hold dual American-Iraqi citizenship.

FRACTIONS PROBLEMS

1. There were 36 coins on the table. 3/4 of them were pennies. 1/3 of them were dimes. 1/3 of them were quarters. How many pennies? Dimes? Quarters? CAN IT BE?

2. Each club member took 24 tickets to sell for the play.
Jack sold 2/3 of his tickets.
Jill sold 3/4 of her tickets.
Dan sold 4/6 of his tickets.
Betty sold 5/8 of her tickets.
Jane sold 7/8 of her tickets. How many did each member sell? CAN IT BE?

3. Cathy broke a window playing ball. She agreed to work 24 hours to pay for the window. She made the following work plan: She planned to work: 1/4 of the 24 hours on Monday,
3/8 of the 24 hours on Tuesday,
2/4 of the hours on Wednesday.
After she works those 3 days, how many more days will she have to work?

4. A DETECTIVE PROBLEM: Jack was supposed to do 19 whole push-ups for exercise. He said he did 2/3 of them before falling on his face. Can it be? Jill said Jack made a mistake in arithmetic. Why did Jill think so?

5. There were 21 horses to be bathed. Mary bathed 1/3 of them. Jill bathed 2/7 of them.
How many did Mary bathe?
How many did Jill bathe?
How many were still dirty?

6. BE SURE YOU LAY OUT THE BLOCKS AND COUNT THEM VERY CAREFULLY FOR THIS PROBLEM: Suppose you have 64 dogs and cats.
3/8 of them are black dogs.
1/4 of them are yellow cats.
4/8 of them are white cats.
The rest are white dogs.
How many are black dogs? Yellow Cats? White cats? White dogs? CAN IT BE?

7. There were 24 airplanes at the airport.
1/4 of them were out of gas.
2/3 of them had flat tires.
1/6 of them had broken propellers, and the rest were o.k.

(a) How many were out of gas? (b) How many had flat tires?
(c) How many had broken propellers? (d) How many were o.k.?
(e) If each plane could carry 10 people, how many people could the o.k. planes carry? CAN IT BE?

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1/3 of them were Mexican,
1/6 of them were African, and the rest were Chinese.
(a) How many were Canadian? (b) Mexican? (c) African? (d) Chinese? CAN IT BE?

9. Mrs. Sanders won $24,000 in the lottery. Let each block represent 1,000 dollars.
She spent 3/8 of it for a car.
She spent 1/4 of it to paint the house.
She put the rest in the bank.
(a) How much did the car cost? (b) The paint job cost how much? (c) How much did she put in the bank? CAN IT BE?

10. Binky invited 12 friends to a party.
1/3 of them came to the party.
1/2 of them ate cake.
4/6 of them ate ice cream. (a) How many came? (b) How many ate cake?
(c) How many ate ice cream? CAN IT BE?

11. Mrs. Smith has 12 children.
1/2 of them want to be fire fighters when they grow up.
1/2 of them want to be writers.
2/3 of them are little girls.
(a) How many want to be fire fighters?
(b) How many want to be writers?
(c) How many are girls?
(d) Can you be sure some of the girls want to be fire fighters?
(e) Can you be sure some of the girls want to be writers? CAN IT BE?

12. 27 peaches grow on your peach tree.
1/3 of them are ripe.
2/3 of them are not ripe.
1/9 of them have holes in them.
(a) How many are ripe? (b) How many are not ripe?
(c) How many have holes? CAN IT BE?

13. There are 30 microscopes in a laboratory.
1/5 of them are for looking at bugs only.
1/2 of them are for looking at germs only.
2/3 of them are for looking at plants only.
(a) How many are for looking at bugs? (b) How many are for looking at germs?
(c) How many are for looking at plants? CAN IT BE?

14. Suppose there are 45 space ships circling the earth,
4/5 of them American,
1/3 of them Russian,
1/3 of them European.
(a) How many are American? (b) How many are European?
(c) How many are Russian? CAN IT BE?

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15. There are 30 students in the first grade.
1/5 of them are girls.
9/10 of them are boys.
1/3 of them walk to school.
(a) How many of them are girls? (b) How many of them are boys?
(c) How many of them walk to school? CAN IT BE?

16. Gerald has 6 children. On his way to work, he drops 2/3 of them off at a nursery school, and 1/2 of them at an elementary school.
(a) How many of Gerald's children go to nursery school?
(b) How many go to elementary school? CAN IT BE?

17. Don had 4 dreams last night.
1/2 were about cars.
3/4 were about his friends.
1/4 were about his parents.
(a) How many were about cars? (b) How many were about his friends?
(c) How many were about his parents? CAN IT BE?

18. Betsy is building a house worth $12,000.
1/2 of that cost is for wood.
1/4 of that cost is for cement.
2/6 of that cost is for metal.
(a) How much money goes to wood? (b) How much money goes to cement?
(c) How much money goes to metal? CAN IT BE?

19. The monster motel houses 35 monsters.
2/5 of them are hairy.
4/7 of them have horns.
1/2 of them have three eyes.
(a) How many of them are hairy? (b) How many of them have horns?
(c) How many of them have three eyes? CAN IT BE?

20. There are 40 bigfoot monsters on earth.
1/4 live in the arctic.
1/2 live at the equator.
2/4 live in the U.S.A.
(a) How many live in the arctic? (b) How many live at the equator?
(c) How many live in the U.S.A? CAN IT BE?

21. There were 36 people on an international committee.
1/3 of them were Americans.
1/4 of them were Iraqis.
1/2 of them were Chinese.
(a) How many of them were Americans? (b) How many of them were Iraqis?
(c) How many of them were Chinese? CAN IT BE?

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LOCUS OF POINTS SECTION

TEACHING SUGGESTIONS FOR THE LOCUS OF POINTS SECTION
Have your students use graph paper to draw figures for the problems. They need compasses and rulers. The geometry concepts needed for these problems are:

1. "Locus" is Latin for place.

2. The locus of points a given distance, D, from a point, P, is a circle with center at P and radius D.

3. The locus of points a given distance, D, from a line, L, is two lines, parallel to and D units from L.

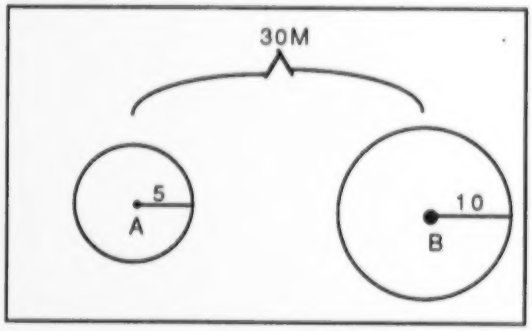
4. The locus of points equidistant from two parallel lines is a line parallel to the given lines and midway between them.

5. The locus of points equidistant from the rays of an angle is the angle-bisector.

6. The locus of points equidistant from two points is the perpendicular bisector of the segment joining the two points.

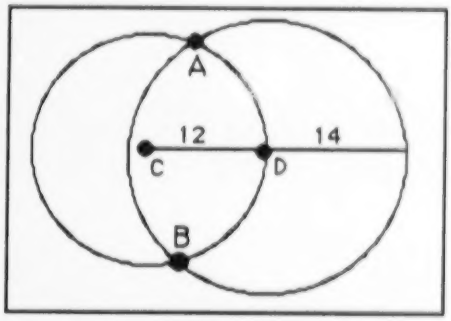
7. For purposes of these problems we define the distance from a point P on the circle to be the perpendicular distance from the tangent at point P.

LOCUS OF POINTS SOLUTIONS



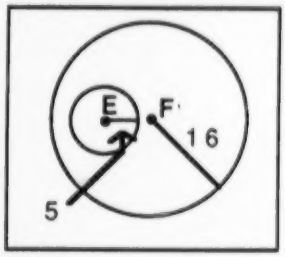
1. Points A and B are 30 meters apart. What is the locus of points which are 5 meters from point A and 10 meters from point B?

SOLUTION: None. The two circles do not intersect.



2. Points C and D are 12 meters apart. What is the locus of points which are 12 meters from point C and 14 meters from point D?

SOLUTION: Two points, A and B. The intersection of a circle with center at C and radius 12, with a circle with center at D and radius 14.

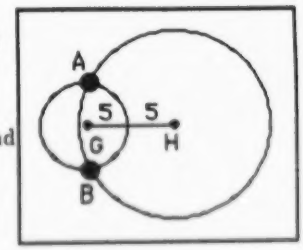


3. Points E and F are 10 meters apart. What is the locus of points which are 5 meters from point E and 16 meters from point F?

SOLUTION: No Solution, the circles do not intersect.

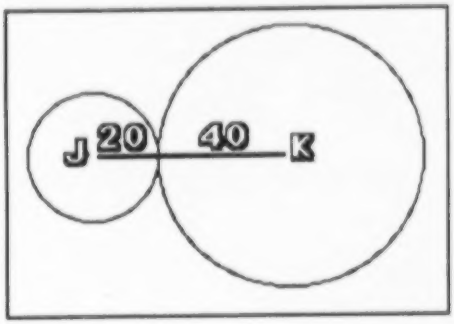
4. Points G and H are 10 meters apart. What is the locus of points which are 5 meters from point G and 12 meters from point H?

SOLUTION: Two points, A and B, the intersection of circle with center at G and radius 5, and the circle with center at H and radius 12.



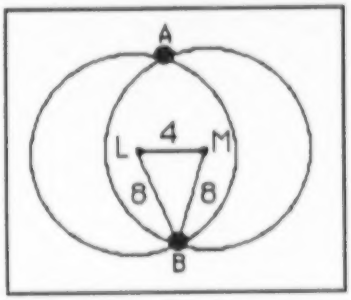
5. Points J and K are 60 miles apart. What is the locus of points which are 20 miles from point J and 40 miles from point K?

SOLUTION: One point, where the circles are tangent.



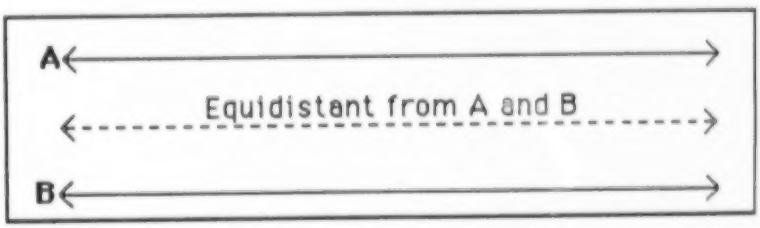
6. Points L and M are 4 meters apart. What is the locus of points 8 meters from point L and from point M.

SOLUTION: Two points, A and B, where the circles intersect.



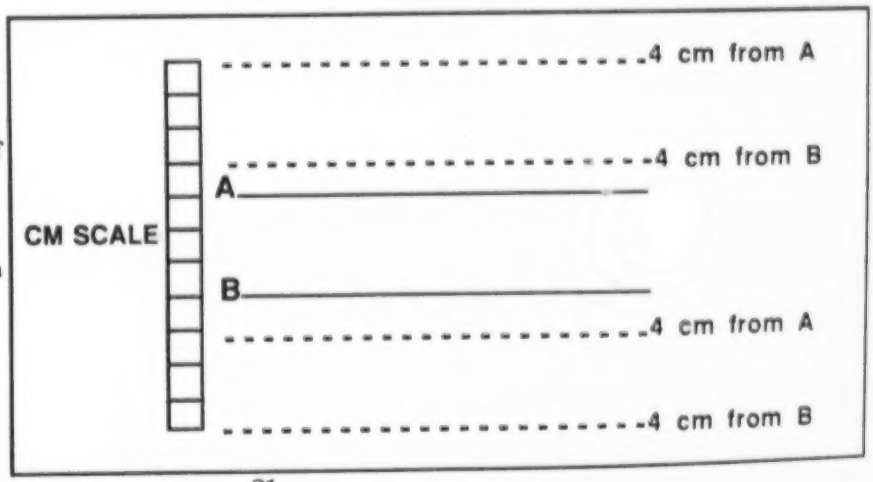
7. Lines A and B are parallel and 3 centimeters apart. What is the locus of points equidistant from lines A and B?

SOLUTION: A line parallel to lines A and B and midway between them.



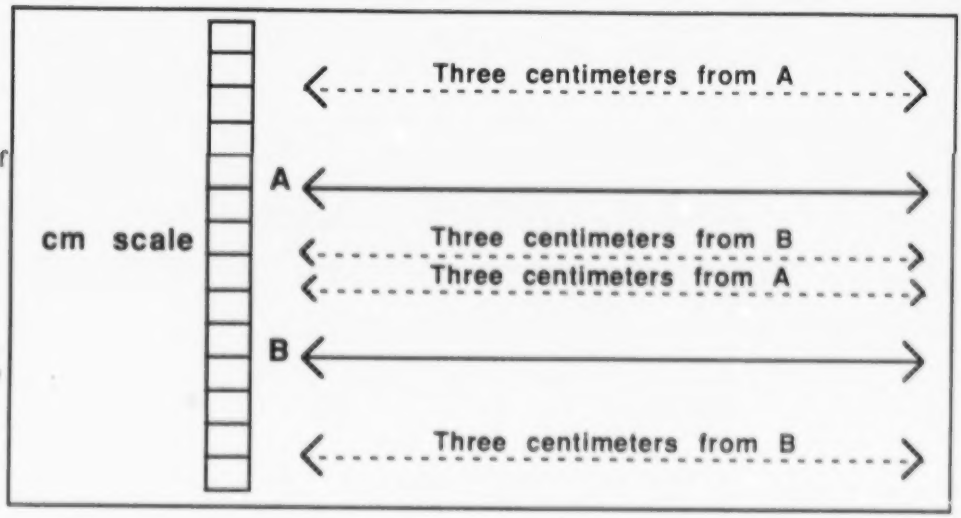
8. Lines A and B are parallel and 3 centimeters apart. What is the locus of points 4 centimeters from line A and 4 centimeters from line B?

SOLUTION: There are no points which lie 4 centimeters from line A and 4 centimeters from line B.



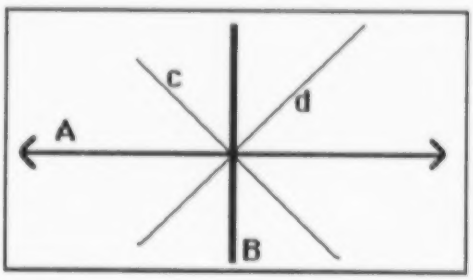
9. Lines A and B are 5 centimeters apart and parallel. What is the locus of points 3 centimeters from line A or 3 centimeters from line B?

SOLUTION: Since the condition is 3 centimeters from line **A or** line B, the solution is made up of the 2 lines 3 centimeters from line A and the 2 lines 3 centimeters from line B.



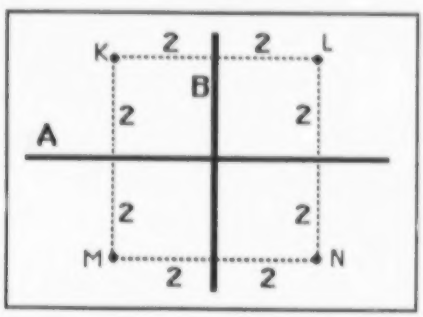
10. Lines A and B intersect at right angles. What is the locus of points equidistant from lines A and B?

SOLUTION: Points on lines C and D are equidistant from lines A and B.



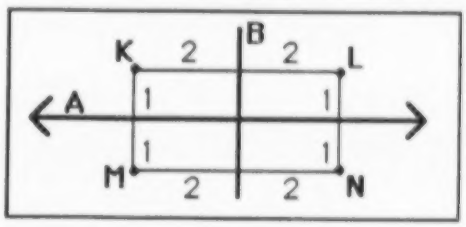
11. Lines A and B intersect at right angles. What is the locus of points 2 centimeters from line A and 2 centimeters from line B?

SOLUTION: Points K, L, M, N are 2 centimeters from lines A and B.



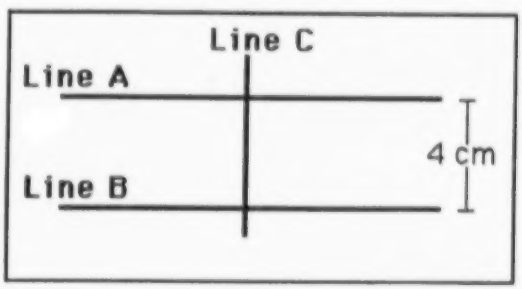
12. Lines A and B intersect at right angles. What is the locus of points one centimeter from line A, two centimeters from line B, and less than one centimeter from where the lines intersect.

SOLUTION: No solution. Points K, L, M, N satisfy the first 2 conditions, but are greater than one centimeter from the intersection.



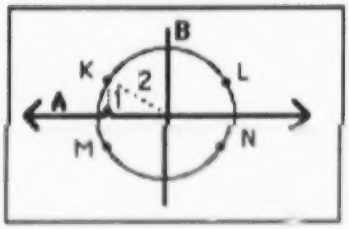
13. Lines A and B are 4 centimeters apart and parallel. Line C intersects lines A and B at right angles. What is the locus of points on line C, 3 centimeters from line A and 3 centimeters from line B?

SOLUTION: No solution. Any point 3 cm from line A is either 1 cm or 7 cm from line B.



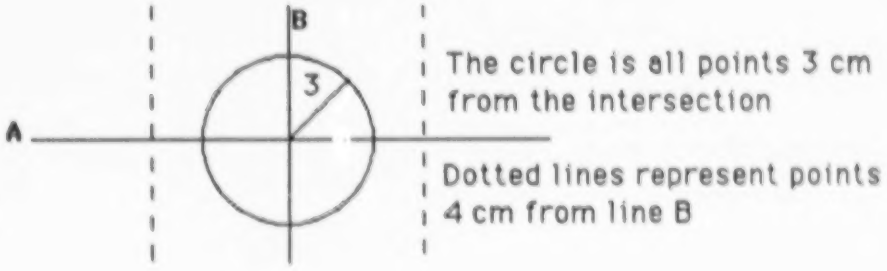
14. Lines A and B intersect at right angles. What is the locus of points 2 centimeters from the point of intersection and one centimeter from line A?

SOLUTION: Points K, L, M, N.



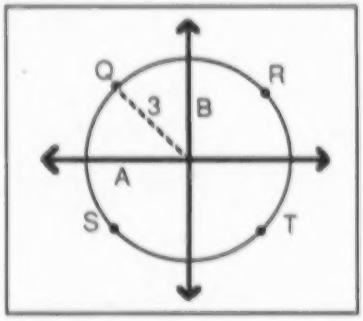
15. Lines A and B intersect at right angles. What is the locus of points 3 centimeters from the point of intersection and 4 centimeters from line B?

NO SOLUTION, points 4 cm from line B are more than 3 cm from the intersection



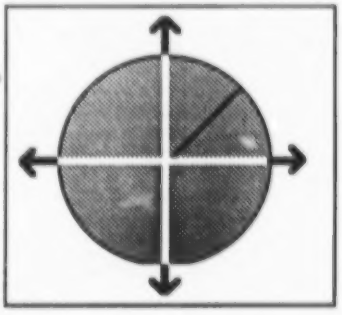
16. Lines A and B intersect at right angles. What is the locus of points 3 centimeters from the point of intersection and equidistant from lines A and B?

SOLUTION: Points Q, R, S, T.



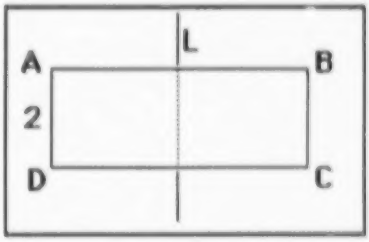
17. Lines A and B intersect at right angles. What is the locus of points less than or equal to 3 centimeters from the intersection of lines A and B?

SOLUTION: All the points inside the circle with center at the intersection and radius 3.



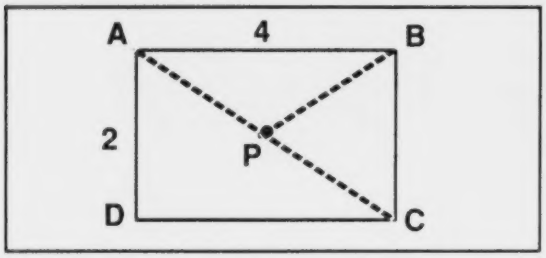
18. Rectangle ABCD is 2 centimeters wide (A to D), 4 centimeters long (A to B). What is the locus of points outside the rectangle equidistant from points A, B and C?

SOLUTION: The points on line L which are outside the rectangle.



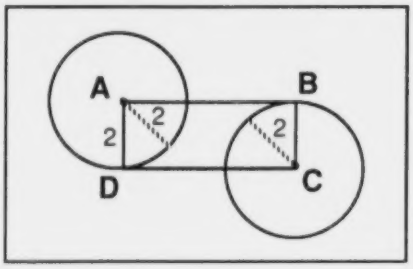
19. Rectangle ABCD is 2 centimeters wide (A to D), 4 centimeters long (A to B). What is the locus of points equidistant from points A, B, and C?

SOLUTION: Point P.

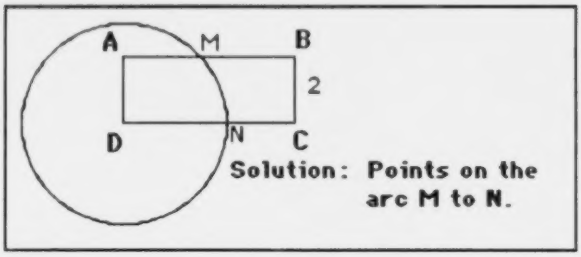


20. Rectangle ABCD is 2 centimeters wide (A to D), 4 centimeters long (A to B). What is the locus of points 2 centimeters from point A and 2 centimeters from point C?

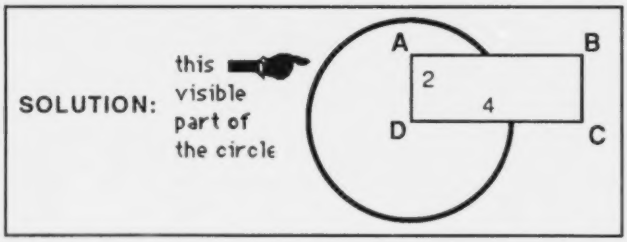
SOLUTION: None. The two circles do not intersect.



21. Rectangle ABCD is 2 centimeters wide (A to D), 4 centimeters long (A to B). What is the locus of points inside the rectangle and 3 centimeters from point D?

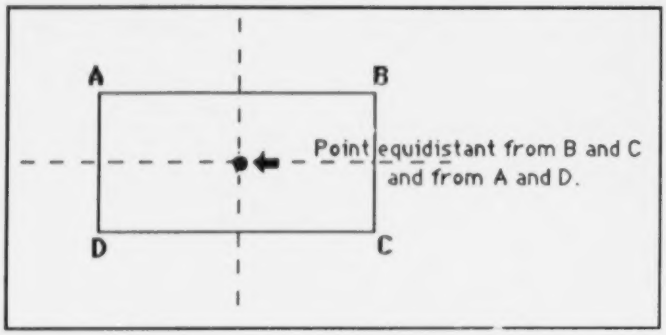


22. Rectangle ABCD is 2 centimeters wide (A to D), 4 centimeters long (A to B). What is the locus of points outside the rectangle and 3 centimeters from point D?



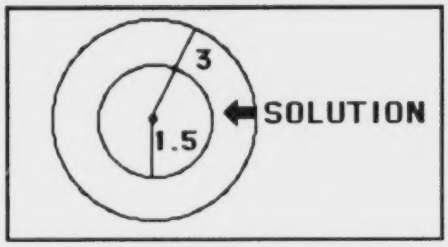
23. Rectangle ABCD is 2 centimeters wide (A to D), 4 centimeters long (A to B). What is the locus of points outside the rectangle and equidistant from points A, B, C, and D?

SOLUTION: None.



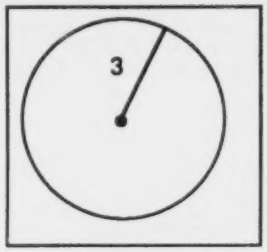
24. Given a circle with radius of 3 centimeters, what is the locus of point equidistant from the circle and from its center?

SOLUTION: A circle with the same center and radius 1.5 centimeters.



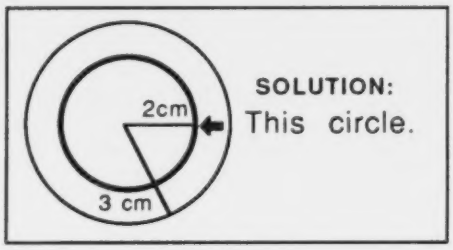
25. Given a circle with radius of 3 centimeters, what is the locus of points outside the circle and 2 centimeters from its center.

SOLUTION: None. Any point outside the circle is more than 2 centimeters from its center.



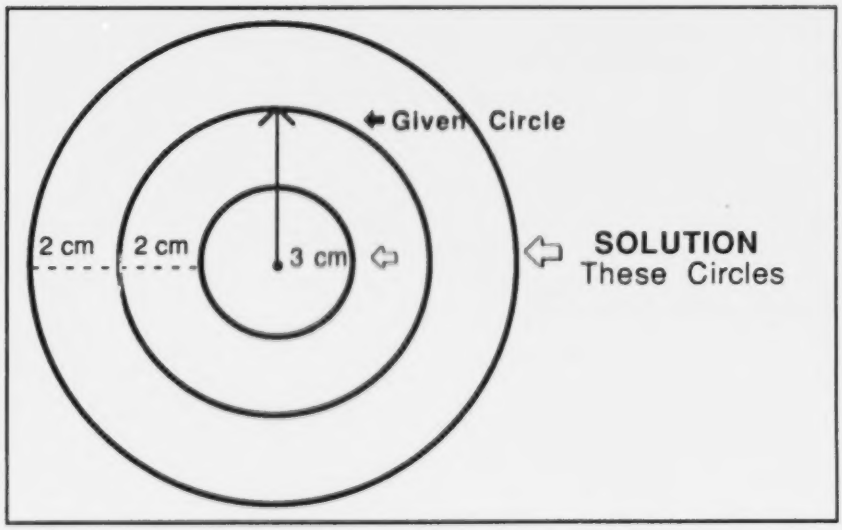
26. Given a circle with radius of 3 centimeters, what is the locus of points inside the circle and 2 centimeters from its center?

SOLUTION: See the diagram.



27. Given a circle with radius of 3 centimeters, what is the locus of points 2 centimeters from the circle?

SOLUTION: See the diagram.



LOCUS OF POINTS PROBLEMS

1. Points A and B are 30 meters apart. What is the locus of points which are 5 meters from point A and 10 meters from point B?

2. Points C and D are 12 meters apart. What is the locus of points which are 12 meters from point C and 14 meters from point D?

3. Points E and F are 10 meters apart. What is the locus of points which are 5 meters from point E and 16 meters from point F?

4. Points G and H are 10 meters apart. What is the locus of points which are 5 meters from point G and 12 meters from point H?

5. Points J and K are 60 miles apart. What is the locus of points which are 20 miles from point J and 40 miles from point K?

6. Points L and M are 4 meters apart. What is the locus of points 8 meters from point L and from point M?

7. Lines A and B are parallel and 3 centimeters apart. What is the locus of points equidistant from lines A and B?

8. Lines A and B are parallel and 3 centimeters apart. What is the locus of points 4 centimeters from line A and 4 centimeters from line B?

9. Lines A and B are 5 centimeters apart and parallel. What is the locus of points 3 centimeters from line A, **or** 3 centimeters from line B?

10. Lines A and B intersect at right angles. What is the locus of points equidistant from lines A and B?

11. Lines A and B intersect at right angles. What is the locus of points 2 centimeters from line A and 2 centimeters from line B?

12. Lines A and B intersect at right angles. What is the locus of points one centimeter from line A, 2 centimeters from line B, and less than one centimeter from where the lines intersect?

13. Lines A and B are 4 centimeters apart and parallel. Line C intersects lines A and B at right angles. What is the locus of points on line C, 3 centimeters from line A and 3 centimeters from line B?

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14. Lines A and B intersect at right angles. What is the locus of points 2 centimeters from the point of intersection and one centimeter from line A?

15. Lines A and B intersect at right angles. What is the locus of points 3 centimeters from the point of intersection and 4 centimeters from line B?

16. Lines A and B intersect at right angles. What is the locus of points 3 centimeters from the point of intersection and equidistant from lines A and B?

17. Lines A and B intersect at right angles. What is the locus of points less than or equal to 3 centimeters from the intersection of lines A and B?

18. Rectangle ABCD is 2 centimeters wide (A to D), 4 centimeters long (A to B). What is the locus of points outside the rectangle, and equidistant from points A, B, and C?

19. Rectangle ABCD is 2 centimeters wide (A to D), 4 centimeters long (A to B). What is the locus of points equidistant from points A, B, and C?

20. Rectangle ABCD is 2 centimeters wide (A to D), 4 centimeters long (A to B). What is the locus of points 2 centimeters from point A and 2 centimeters from point C?

21. Rectangle ABCD is 2 centimeters wide (A to D), 4 centimeters long (A to B). What is the locus of points inside the rectangle and 3 centimeters from point D?

22. Rectangle ABCD is 2 centimeters wide (A to D), 4 centimeters long (A to B). What is the locus of points outside the rectangle and 3 centimeters from point D?

23. Rectangle ABCD is 2 centimeters wide (A to D), 4 centimeters long (A to B). What is the locus of points outside the rectangle and equidistant from points A, B, C, and D?

24. Given a circle with radius of 3 centimeters, what is the locus of points equidistant from the circle and from its center?

25. Given a circle with radius of 3 centimeters, what is the locus of points outside the circle and 2 centimeters from its center?

26. Given a circle with radius of 3 centimeters, what is the locus of points inside the circle and 2 centimeters from its center?

27. Given a circle with radius of 3 centimeters, what is the locus of points 2 centimeters from the circle?

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INVERSE OPERATIONS

TEACHING SUGGESTIONS FOR THE INVERSE OPERATIONS SECTION

The inverse operations section consists of 4 problem sets, all of which are intended to be performed with a calculator.

Set 1 is a preparatory set, with 10 problems in using the calculator to perform a string of operations. (Since there is little thought required by these ten tasks, they would more properly be called "exercises").

Set 2 is also preparatory. It eases the student into inverse operations problems with ten two-step and ten three-step problems, all of which do have solutions.

Set 3 is the focal set for this section. It contains 25 problems, some of which do not have solutions.

The students need the following definitions:

The natural numbers are {1, 2, 3, . . .}
The whole numbers are {0, 1, 2, 3, . . .}
The integers are {. . ., -3, -2, -1, 0, 1, 2, 3, . . .}
The rational numbers are: Any number which can be written in the form A/B, with A and B integers. Notice that the definition says "<u>can</u> be written in the form A/B." That makes all integers rational, since they can be written in that form.

Notice that the "started-with" sets (column 1, set 3) are given as a span of numbers (we avoid the word "range" here, since the range of an algebraic function is the output). This allows you to start students on the problems using the guess-and-check method. Starting with that method is strongly recommended because it helps to develop students' quantitative sense for the operations. Some students will discover for themselves that they can start with the output-number and do the inverse operations to arrive at the input. It is recommended that a student solve at least ten problems by guess-and-check before being taught to do the inverse operations.

Note regarding problems 10 and 15: These problems will raise the issue of division by zero. Students who are aware that division by zero produces no result are likely to come to the point in the operations string where multiplication by zero occurs, and declare that the problem has no solution because they cannot divide by zero. However, we must be careful in stating the question. These two problems do have solutions if the question is "Can you start with the designated input and arrive at the output-number?" The answer to that question is yes, since any number can be used as the input to arrive at the given output. What needs to be stressed is that an operation or a rule which selects <u>everything</u>, in fact, selects <u>nothing</u>. It may help to illustrate the point if you ask what the name "Billy" means if everyone in the world had that name.

Set 4 consists of ten problems involving squares and square roots.

INVERSE OPERATIONS ANSWER KEY

SET ONE (output)	SET TWO (input)	SET THREE (input) (can it be?)	SET FOUR (input) (can it be?)
1. 9	1. 4	1. 3 Yes	1. 4 Yes
2. 18	2. 1	2. 18 Yes	2 .25 No
3. 50	3. 400	3. -2 No	3. 144 Yes
4. 2	4. 25	4. 6.5 No	4. 0 No
5. 32	5. 8	5. 7.5 Yes	5. Any number Yes or No, (see teaching suggestions)
6. 100	6. 5	6. -12 Yes	
7. 1	7. 8	7. 12.5 No	6. 16 No
8. 0	8. 60	8. .5 No	7. 100 Yes
9. 10	9. 800	9. 0 No	8. 1/4 Yes
10. 500	10. 0	10. Any number Yes or No, (see teaching suggestions)	9. 1/100 No
	11. 12		10. 1 Yes
	12. 75	11. 6 Yes	
	13. 75	12. 0 No	
	14. 18	13. -5 No	
	15. 0	14. -6 Yes	
	16. 1000	15. Any number Yes or No, (see teaching suggestions)	
	17. 1000		
	18. 65	16. 0 Yes	
	19. 10	17. -8 No	
	20. 6	18. -100 Yes	
		19. 10 Yes	
		20. 0.5 No	
		21. 1.5 No	
		22. -10 Yes	
		23. 8 No	
		24. 0 Yes	
		25. 0 No	

INVERSE OPERATIONS PROBLEMS-SET #1

USE A CALCULATOR TO FIND THE OUTPUT NUMBER

	INPUT	OPERATION	OUTPUT
1	6	+8 x2 -1 ÷3	
2	10	-3 x6 ÷21 x9	
3	-2	x5 +20 x10 ÷2	
4	25	÷5 x6 ÷5 -4	
5	100	-40 ÷15 x4 +16	
6	40	x2 ÷40 +8 x10	
7	0	x65 ÷8 +6 ÷6	
8	1	x65 +15 ÷20 -4	
9	5	÷5 +5 ÷3 x5	
10	1000	÷100 ÷10 x1000 -500	

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INVERSE OPERATIONS PROBLEMS-SET #2

USE A CALCULATOR TO FIND THE INPUT NUMBER

	INPUT	OPERATION	OUTPUT
1		x3 +8	20
2		x18 +9	2
3		-250 +15	10
4		+10 x2	70
5		+2 x7	28
6		x5 +5	30
7		x2 +8	2
8		+15 +15	19
9		-300 +100	5
10		+25 x5	125
11		-2 x2 +10	2
12		+25 x3 +3	3

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USE A CALCULATOR TO FIND THE INPUT NUMBER

	INPUT	OPERATION	OUTPUT
13		-25 ÷2 +25	50
14		x2 ÷6 +20	26
15		x1000 +2 x8	16
16		÷4 -50 x4	800
17		-500 x2 ÷500	2
18		÷65 x40 -20	20
19		x10 ÷100 +8	9
20		+24 x2 ÷15	4

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INVERSE OPERATION PROBLEMS-SET #3

DEFINITIONS

Counting Numbers: {1, 2, 3, 4, ...}
Whole Numbers: {0, 1, 2, 3, 4, ...}
Integers: {...-4, -3, -2, -1, 0, 1, 2, 3, 4,...}
Rational Numbers: Any number which can be written in the form A/B, with A and B integers. Notice that the definition says "can be written in the form A/B." That makes all integers rational, since they can be written in that form. So, 17, 365, 0, 2,734,560, - 79, -1000 are all rational.

USE THE CALCULATOR TO FIND THE INPUT NUMBER

	STARTED WITH	INPUT	OPERATIONS	OUTPUT	CAN IT BE?
1	A whole number 0 to 15		+5 x3 -6 x2	36	
2	A whole number 10 to 25		-3 x2 +10 x5	200	
3	A whole number 0 to 10		+8 x6 -4 +4	8	
4	A whole number 0 to 10		x2 +5 -8 +10	20	
5	A positive rational number 6 to 8		+0.5 -7 x50 +25	2	
6	An integer -15 to 0		+20 +5 x10	16	
7	An integer 1 to 15		x10 -25 +5 +5	25	

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USE A CALCULATOR TO FIND THE INPUT NUMBER

	STARTED WITH	INPUT	OPERATIONS	OUTPUT	CAN IT BE?
8	An integer 0 to 10		+3.5 x8 -2 +15	45	
9	A negative number -10 to -1		+5 ÷5 -1 +6	6	
10	A rational number 0 to 15		+5 x0 +6 ÷2	3	
11	A natural number 0 to 15		x12 -36 ÷36 +12	13	
12	A natural number 1 to 15		÷3 +84 ÷4 +3	7	
13	A natural number 1 to 15		+10 x50 -125 +25	150	
14	Negative rational -15 to -1		+4 -4 x6	-36	
15	Whole number -15 to -1		+10 ÷2 x0 +1	1	
16	A rational number 0 to 15		+5 x8 ÷10 -2	2	

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USE A CALCULATOR TO FIND THE INPUT NUMBER

	STARTED WITH	INPUT	OPERATIONS	OUTPUT	CAN IT BE?
17	A natural number 1 to 15		x10 +85 ÷5 x(-8)	-8	
18	A negative integer -110 to 0		÷2 ÷2 x4 +50	-50	
19	An integer 0 to 15		-5 +15 ÷4 x0.5	2.5	
20	An integer 0 to 15		x2 x20 x0.5 ÷2	5	
21	A natural number 0 to 15		x6 ÷3 +17 x2.5	50	
22	A rational number -15 to 15		x2 +40 x.75 ÷5	20	
23	A negative integer -15 to -1		÷8 ÷0 .5 -30 ÷2	1	
24	A whole number 0 to 15		÷1 ÷1 x1 +1	2	
25	A natural number 1 to 15		÷8 ÷4 x6 -6	6	

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INVERSE OPERATIONS PROBLEMS-SET #4

USE A CALCULATOR TO FIND THE INPUT NUMBER

	STARTED WITH	INPUT	OPERATIONS	OUTPUT	CAN IT BE?
1	An integer		square -6 x10 take square root	10	
2	A whole number		take square root ÷5 x5	0 .5	
3	An integer		take square root x12 ÷2 -12	60	
4	A natural number		+20 -4 take square root add 1	5	
5	A rational number		take square root x0 +12 x10	120	
6	A negative integer		take square root x60 -204 take square root	6	
7	An integer		take square root x10 -50 ÷5	10	
8	A rational number		take square root added 1/2 x60 +30	2	
9	A whole number		take square root x10 +8	9	
10	A whole number		square +3 take square root -2	0	

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